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A Geo/Geo/1 Discrete Time Inventory Model with Service Time

Anilkumar M.P [1] and K. P. Jose [2]
[1] *Department of Mathematics, T.M. Govt. College, Tirur-676502, Kerala, India*
[2] *PG & Research Department of Mathematics, St. Peter's College, Kolenchery-682311, Kerala, India*
Email: anilkumarmp77@gmail [1]; kpjspc@gmail.com [2]

Abstract-This paper considers a Geo/Geo/1 discrete time inventory model with service time. Arrival of customers form Bernoulli process and service time is geometrically distributed. Assuming zero lead time, we obtained a closed form solution for the steady state.Matrix analytic method is used to find the steady state probability vectors. We calculated some performance measures and constructed a suitable cost function. The optimum value of cost function is also obtained.

Index Terms – Discrete-Time, Inventory, Geometric Distribution, Matrix Analytic Method

1. INTRODUCTION

Almost all queueing models in literature before 1990 were discussed on the basis of continuous time Markov chain. The only models such as M/G/1 and GI/M/1 were studied on the basis of embedded Markov chain as discrete time models. Such kind of models were discussed in Kendal [7] and [8]. The other discrete time models later studied by Galliher and Wheeler [6]. Defermos and Neuts [4] discussed a basic example of a single server queue in discrete time and analyzed the time dependent behaviour of the queue in terms of a bivariate Markov chain. Neuts and Klimko [9] examined the numerical problems arising in the computation of higher order moments of the busy period for certain classical queues of the M/G/I type, both in discrete and in continuous time. A systematic study discrete time queueing models were done in Alfa [1]. The advantages and disadvantages of discrete time analysis is also discussed in this paper. The paper focuses on setting up several queueing systems as discrete time quasi-birth and death processes and then shows how to use Matrix Geometric Method (MGM) to analyze the problems.A detailed discussion of Matrix Analytic Method in discrete time Markov chain is done in [2] and [3].

This paper is an extension of a work in [5]. We study a discrete time inventory system in which arrival occurs according to a Bernoulli process with parameter p and service time is geometrically distributed with parameter q. Assume that lead time is zero and no customers are allowed to enter in the system when the inventory level is zero. We use Matrix Analytic Method [10] to find the steady state probability vectors.

The rest of the paper is organized as follows. Section 2 provides mathematical modeling and transition analysis. Stability condition and steady state probability vector are derived in section 3 and 4 respectively. Relevant performance measures are included in section 5. Finally, Section 6 provides numerical experiments.

2. MATHEMATICAL MODELING AND ANALYSIS

Following notations and assumptions are used in this paper.

Assumptions

i. Maximum inventory level is S
ii. Inter-arrival time is geometrically distributed with parameter p
iii. Lead time is zero
iv. Service time is geometrically distributed with parameter q.
v. Any demand require at least one unit of service time (late arrival with delayed access).
vi. Replenishment takes place at the end of slot boundaries.

3. Notations

i. $I(n)$: Inventory level at an epoch $n.$
ii. $N(n)$: Number of customers at time n
iii. $\bar{a} = 1 - a$ where $0 \leq a \leq 1$
iv. **e** : Column vector of 1's of appropriate order

Now $\{(N(n), I(n)), n = 1,2,3 \ldots\}$ is a Level Independent Quasi-Birth Death process (LIQBD) on the state space $\{(i, j); 0 \leq i < \infty, 1 \leq j \leq S\}$. The transition probability matrix P of this process is given by

$$= \begin{bmatrix} B_1 & B_0 & 0 & 0 & \ldots & 0 \\ A_2 & A_1 & A_0 & \ldots & \ldots & 0 \\ 0 & A_2 & A_1 & A_0 \ldots & \ldots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \\ \vdots & \vdots & \vdots & \ldots & \ldots & \end{bmatrix}$$

where the elements of P are square matrices of order S and are given by

$$B_1 = \bar{p} I_S$$

$$B_0 = pI$$

$$[A_2]_{i,j} = \begin{cases} \bar{p}q & if\, i = 1, j = S\,; \\ \bar{p}q & if\, i = j+1, j < S \\ 0 & otherwise \end{cases}$$

$$[A_1]_{i,j} = \begin{cases} \bar{p}\bar{q} & if\, i \geq 1, j = S\,; \\ pq & if\, i = s, j = S \\ pq & if\, i = j+i, j < s \\ 0 & otherwise \end{cases}$$

$$[A_0]_{i,j} = \begin{cases} p\bar{q} & if\, i = j+1, j < S \\ 0 & otherwise \end{cases}$$

4. STABILITY CONDITION

For the studying stability condition, consider the matrix

$$A = A_0 + A_1 + A2$$

$$A = \begin{bmatrix} \bar{q} & 0 & 0 & \dots & q \\ q & \bar{q} & 0 & \dots & 0 \\ 0 & 0 & q & \bar{q}\dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ \vdots & \vdots & \vdots & q & \bar{q} \end{bmatrix}$$

The given markov chain is stable if and only if

$$\boldsymbol{\pi} A_0 \boldsymbol{e} < \boldsymbol{\pi} A_2 \boldsymbol{e} (1)$$

where $\boldsymbol{\pi}$ satisfies $\boldsymbol{\pi} A = \boldsymbol{\pi}$ and $\boldsymbol{\pi e} = 1$. (2)

One can refer Neuts [10] for the details of stability condition.On solving(2), we have

$$\boldsymbol{\pi} = \left(\frac{1}{S}, \frac{1}{S}, \dots\dots, \frac{1}{S}\right) = \frac{1}{S}\boldsymbol{e}'$$

Now condition (1) reduces to

$$p\bar{q} < \bar{p}q$$

This implies that system is stable if and only if

$$p < q$$

5. STEADY STATE ANALYSIS

To find the steady state probability vector of P, first assume that service time is negligible. Then thetransition probability matrix becomes

$$\bar{P} = \begin{bmatrix} \bar{p} & 0 & 0 & \dots & p \\ p & \bar{p} & 0 & \dots & 0 \\ 0 & 0 & p & \bar{p}\dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ \vdots & \vdots & \vdots & p & \bar{p} \end{bmatrix}$$

Let $\bar{\boldsymbol{\pi}} = (\pi_s, \pi_{2,\dots\dots,}\pi_S)$ be the steady state probability of $\bar{P}$

Then $\bar{\boldsymbol{\pi}} P = \bar{\boldsymbol{\pi}}$ and $\sum_{i=s}^{S} \pi_i = 1$

This condition leads to

$$\pi_i = \frac{1}{S}, i = 1,2,\dots,S$$

Let $\mathbf{x} = (x_0, x_1, \dots.)$ be the steady state probability vector of P

By looking through the structure of transition probability matrix of the original matrix one can assume that

$$x_1 = k\rho\bar{\boldsymbol{\pi}},\ \ x_i = r^{i-1}x_1, i = 2,3,\dots\dots(3)$$

Then $\mathbf{x}P = \mathbf{x}$ leads to

$$x_0 B_0 + x_1 A_2 = x_0$$

$$x_0 B_1 + x_1 A_1 + x_2 A_2 = x_1$$

$$x_{i-1} A_0 + x_i A_1 + x_{i+1} A_2 = x_i$$

Using (3) in above, we get

$$k\left(\frac{1}{S}\bar{p}, \frac{1}{S}\bar{p}, \dots.\right) + k\rho\left(\frac{\bar{p}q}{S}, \frac{\bar{p}q}{S}, \dots.\right) = k\left(\frac{1}{S}, \frac{1}{S}, \dots\dots\right)$$

$$\rho\frac{\bar{p}q}{S} = \frac{p}{S}$$

Hence, $\rho = \frac{p}{\bar{p}q}$. Again one can that

$$r = \frac{p\bar{q}}{\bar{p}q}$$

Therefore, the components of steady state probability vector of P are given by

$$x_0 = k\bar{\pi},$$

$$x_i = k\frac{p}{\bar{p}q}\left(\frac{p\bar{q}}{\bar{p}q}\right)^{i-1}\bar{\pi}, i = 1,2,3,\dots$$

Now,$\sum_{i=0}^{\infty} x_i \mathbf{e} = 1$ implies that

$$k = 1 - \frac{p}{q}, \text{ provided } \frac{p}{q} < 1$$

Therefore, (2)

$x_i = \left(1 - \frac{p}{q}\right)\left(\frac{p}{\bar{p}q}\right)\left(\frac{p\bar{q}}{\bar{p}q}\right)^{i-1}\left(\frac{1}{S}\right)\mathbf{e}', i \geq 1$ and

and $x_0 = \left(1 - \frac{p}{q}\right)\bar{\pi}$

6. SYSTEM PERFORMANCE MEASURES

Relevent system performance measures are derived.

1) Expected number of customers in the system,EC, is given by

$$EC = \sum_{i=1}^{\infty}\sum_{j=1}^{S} i x_{ij} = \frac{p(1-p)}{q-p}$$

2) Expected inventory level, EIL, is given by

$$EIL = \sum_{i=1}^{\infty}\sum_{j=1}^{S} j x_{ij} = \frac{S+1}{2}$$

3) Expected reorder rate ,ER, is given by

$$ER = q\sum_{i=1}^{\infty} x_{i1} = \frac{p}{S}$$

4) Expected departure after completing the service, ED, is given by

$$ED = q\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} x_{ij} = p$$

7. NUMERICAL EXPERIMENTS

We define expected total cost (ETC) per unit time as

$$ETC = (C_0 + SC_1)ER + C_2EI + C_3EC + (C_4 - C_5)ED + C_6x_0\mathbf{e} \quad (4)$$

where

C_0: The setup cost/order
C_1: Procurement cost/unit
C_2: Inventory holding cost/unit/unit time
C_3: Holding cost of customers/unit/unit time
C_4: Service cost/unit/unit time
C_5: Revenue due to service/unit/unit time
C_6: Running cost of the empty system/unit time

Now (4) becomes,

$$ETC = (C_0 + C_1S)\frac{p}{S} + C_2\left(\frac{S+1}{2}\right) + C_3\frac{p(1-p)}{q-p} + (C_4 - C_5)p + C_6\left(1 - \frac{p}{q}\right)$$

Graphical Illustrations

We draw the graphs of ETC by varying the parameters p and q and keeping other parameters fixed. From figures 1 & 2, one can observe that the minimum values of ETC are 1.55 and 1.47 at $p = 0.74$ and $q = 0.84$ respectively.

$$S = 5; C_0 = 2; C_1 = 12; C_2 = 1; C_3 = 0.1; C_4 = 5; C_5 = 20; C_6 = 2; q = 0.8$$

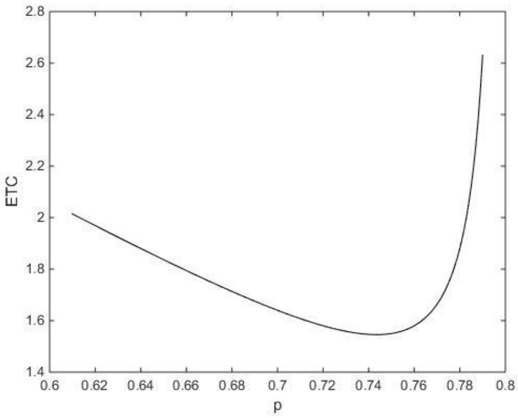

Fig.1 $\boldsymbol{p}$ vs $\boldsymbol{ETC}$

$$S = 5; C_0 = 2; C_1 = 12; C_2 = 1; C_3 = 0.1; C_4 = 5; C_5 = 20; C_6 = 2; p = 0.75;$$

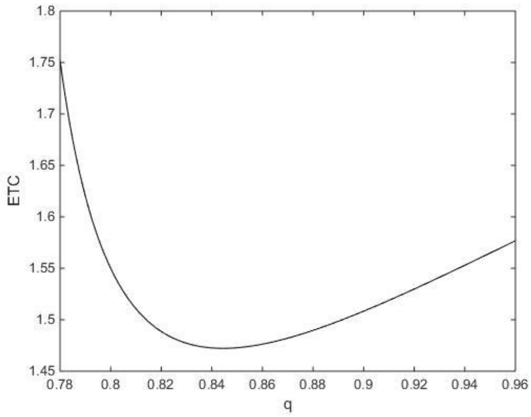

Fig.2 $\boldsymbol{q}$ vs $\boldsymbol{ETC}$

8. CONCLUDING REMARKS

In this paper, we considered a discrete time queueing inventory model with service time. This is analyzed using Matrix Analytic method. A suitable cost function is defined on the basis of relevant performance measures of the system.The optimum value of the cost function is obtained numerically. One can extend the research work by considering discrete phase type service time and Markovian arrival process.

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